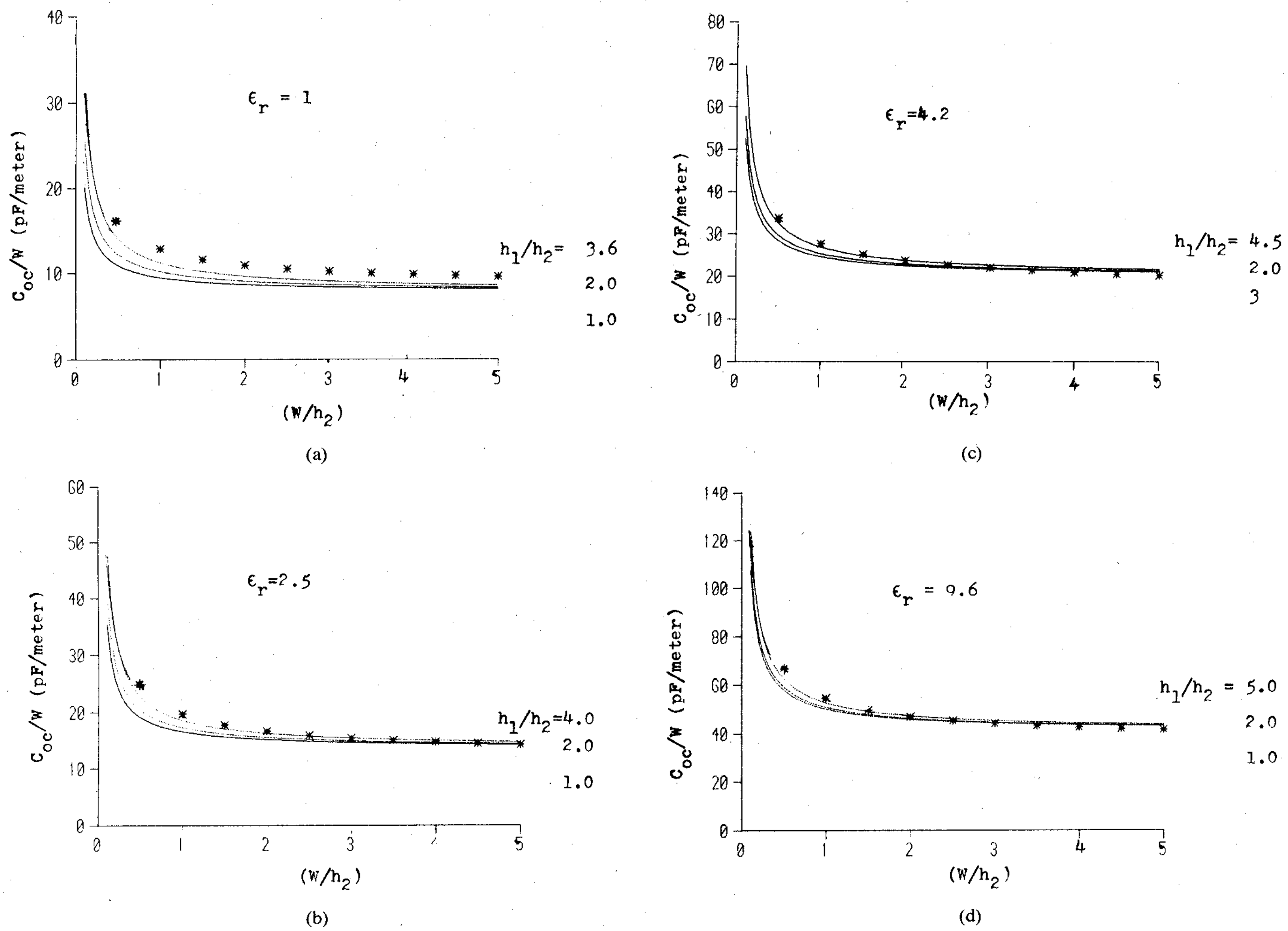

Fig. 3. (a) Open-end capacitance of shielded microstrip and the capacitance of the unshielded microstrip. * * *: obtained from [4]. (b) Open-end capacitance of shielded microstrip and the capacitance of the unshielded microstrip. * * *: obtained from [4]. (c) Open-end capacitance of shielded microstrip and the capacitance of the unshielded microstrip. * * *: obtained from [4]. (d) Open-end capacitance of shielded microstrip and the capacitance of the unshielded microstrip. * * *: obtained from [4].

tance for a semi-infinite plate between ground planes

$$C_{oci} = 0.441 h_i C_i, \qquad i = 1,2. \tag{4}$$

The equivalent additional length Δl_{oc} for the open-end discontinuity in the shielded microstrip may be written as follows:

$$\frac{\Delta l_{oc}}{h_2} = \frac{C_{oc}}{h_2(C_1 + C_2)}. \tag{5}$$

III. Results and Conclusion

The variation of the fringe capacitance at the end of the shilded microstrip with the width-to-height ratio W/h_2 and the shield heights ratio h_1/h_2 using (1) and (3) for various substrate dielectric materials with $\epsilon_r = 1$, 2.5, 4.2, and 9.6 are shown in Fig. 3(a)–(d), respectively.

The results are compared with the limit case of the unshielded microstrip [4]. As the shield heights ratio h_1/h_2 increases, the values of the end fringe capacitance C_{oc} approach those of the unshielded microstrip.

References

[1] S. S. Bedair and M. I. Sobhy, "Accurate formulae for the computer-aided-design of shielded microstrip circuits," *Proc. Inst. Elec., Eng.*, pt. H, MOA, pp. 305–308, Dec. 1980.

[2] H. M. Altschuler and A. A. Oliner, "Discontinuities in the center conductor of symmetric strip transmission line," *IRE Trans. Microwave Theory Tech.*, vol. MTT-8, pp. 328–339, May 1960.

[3] S. B. Cohn, "Shielded coupled-strip line," *IRE Trans. Microwave Theory Tech.*, vol. MTT-3, pp. 29–38, Oct. 1955.

[4] P. Silvester and P. Benedek, "Equivalent capacitances of microstrip open circuits," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-20, pp. 511–516, Aug. 1972.

Extension of an Old Circulator Model

GYULA VESZELY

***Abstract*—An old circulator model consists of an ideal circulator with parallel coupled resonant circuits. This paper determines the parameters of this model at frequencies different from the resonant one. As a conse-**

Manuscript received October 29, 1980; revised March 12, 1981.

The author is with the Department of Theoretical Electricity, Technical University, 1521 Budapest, Hungary.

0018-9480/81/1000-1109$00.75 ©1981 IEEE

quence a unified treatment of the stripline and lumped element circulators is possible.

I. Introduction

An old circulator model is an ideal circulator with parallel coupled resonant circuits [1], [2]. A quantitative model was given in [3], but it was proved only for lumped element circulators. Contrary to [3] we will neglect the frequency dependence of the Polder tensor elements.

II. The Impedance Matrix and Its Equivalent Network

The impedance matrix of symmetric three-port circulators:

$$\boldsymbol{Z}=\begin{bmatrix} Z_1 & Z_2 & Z_3 \\ Z_3 & Z_1 & Z_2 \\ Z_2 & Z_3 & Z_1 \end{bmatrix} \tag{1}$$

where

$$\begin{aligned} Z_1 &= Z^+ + Z^- \\ Z_2 &= Z^+ e^{-j2\pi/3} + Z^- e^{j2\pi/3} \\ Z_3 &= Z^+ e^{j2\pi/3} + Z^- e^{-j2\pi/3}. \end{aligned} \tag{2}$$

Regarding the symmetry the in-phase impedance is zero. This is necessary to the existence of the following model.

As it can be easily seen, if the relations

$$z=j2\sqrt{3}\,\frac{Z^+Z^-}{Z^- - Z^+}, \qquad \text{i.e., } \frac{1}{z}=\frac{1}{j2\sqrt{3}}(Y^+ - Y^-) \tag{3}$$

$$Y=\frac{1}{6}\,\frac{Z^+ + Z^-}{Z^+Z^-}, \qquad \text{i.e., } Y=\frac{1}{6}(Y^+ + Y^-) \tag{4}$$

are fulfilled, the impedance matrix of the network in Fig. 1 is the same as (1), (2). The network in Fig. 2 has the same impedance matrix too, and it is more suitable for practical calculations.

The main results of this paper are the expressions (3), (4), referring to the well-known networks in Figs. 1 and 2.

III. The $Y^\pm$ Admittance Parameters in Two-Mode Approximation

For stripline circulators the network in Fig. 3 is valid, where $L^\pm$ and C can be found in [4], $Q_C = 1/\tan\delta$, where δ is the dielectric loss angle of the ferrite and by a perturbational calculus

$$Q^\pm = \frac{2}{\left(1 \mp \frac{\kappa'}{\mu'}\,\frac{1}{x_{11}^2-1}\right)^2} \cdot \frac{1}{\frac{\mu''^2-\kappa''^2}{\mu_0\mu''}} \cdot \frac{1 \pm \frac{\kappa'}{\mu'}\,\frac{1}{x_{11}^2-1}}{1 \pm \frac{\kappa''}{\mu''}\,\frac{2}{x_{11}^2-1}} \tag{5}$$

where μ' and κ' are the real parts, $-\mu''$ and $-\kappa''$ are the imaginary parts of the Polder tensor elements, μ_0 is the vacuum permeability and $x_{11}=1.84$.

We want, that the network in Fig. 3 be valid for lumped element circulators too. To achieve this it is suitable to draw in the C_{ex} external tuning capacity into $Y^\pm$ as in Fig. 4. Consequently the elements in Fig. 3 for lumped element circulators are $L^\pm$ can be found in [4], $C=3C_{ex}$, $Q_C = 1/\tan\delta$, and $Q^\pm = (\mu^\pm)'/(\mu^\pm)''$.

IV. Results

The network in Fig. 2 is so simple, that closed form expressions can be obtained for the scattering matrix elements. From these expressions the following is derived.

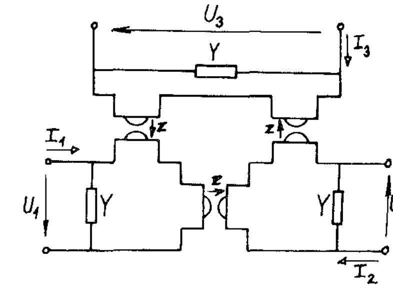

Fig. 1. Network model of circulator.

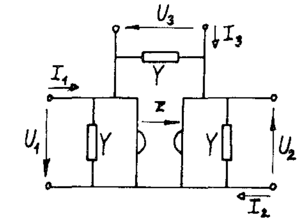

Fig. 2 Equivalent network model.

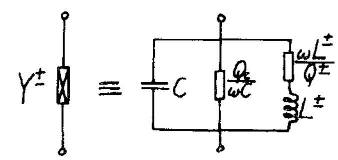

Fig 3. $Y^\pm$ admittances.

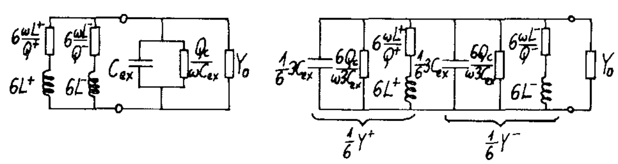

Fig. 4. Drawing the C_{ex} external tuning capacity into $Y^\pm$

a) The relative bandwidth belonging to S_{11} reflection is in the lossless case

$$\omega=\frac{2\sqrt{3}\,|S_{11}|\,f(\eta)}{\sqrt{1+\frac{3}{4}[f(\eta)]^2}} \tag{6}$$

where $\eta=\kappa'/\mu'$ and $f(\eta)=(L^+ - L^-)/(L^+ + L^-)$.

b) The bandwidth decreases under the effect of losses, although this decreasing is very small (in typical cases 1–2 percent).

c) The forward transmission loss, in decibels, is

$$|S_{13}| \approx 2.51\,\frac{1}{f(\eta)Q_p} \tag{7}$$

where

$$Q_p = \left(1/Q^+ + 1/Q^- + 2/Q_C\right)^{-1}. \tag{8}$$

For lumped element circulators if $Q^+ = Q^-$, then (8) is the same as Konishi's approximate result [1]. If $Q^+ \neq Q^-$, then (8) is the correct expression.

References

[1] Y Konishi, "Lumped element Y circulator," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-13, pp. 852–864, Nov. 1965.

[2] H. Bosma, "A general model for junction circulators; choice of magnetization and bias field," *IEEE Trans. Magnetics*, vol. MAG-4, pp. 587–596, Sept 1968.

[3] J. O. Bergman and C. Christensen, "Equivalent circuit for a lumped element Y circulator," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-16, pp. 308–310, May 1968.
[4] G. Bittar and Gy. Veszely, "A general equivalent network of the input impedance of symmetric three-port circulators," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-28, pp. 807–808, July 1980.

First-Order Bragg Interactions in a Gyromagnetic-Dielectric Waveguide

MAKOTO TSUTSUMI, MEMBER, IEEE

***Abstract*—First-order Bragg interactions in a gyromagnetic-dielectric waveguide are investigated theoretically. With the aid of a singular perturbation procedure the coupled mode equations governing the nature of transverse electric wave interactions are derived. Bragg reflection characteristics are shown numerically as a function of the magnetic field.**

I. Introduction

Bragg interaction in a planer dielectric waveguide whose properties vary periodically is very interesting topic from both practical and theoretical points of view [1]. Recently Seshadri has investigated asymmetric first-order Bragg interactions in a active dielectric waveguide by a singular perturbation procedure using multiple space scales [2]. The author has also studied the reflection of millimeter wave by a corrugated dielectric slab using a singular boundary procedure, and has confirmed the theoretical results by experiments [3].

This short paper investigates Bragg reflection characteristics in a gyromagnetic-dielectric waveguide whose refractive indexes vary sinusoidally in the direction of the wave propagation. Reflection characteristics in such a waveguide will be more sensitive to the change of the magnetic field than that of a corrugated gyromagnetic slab [4]. With the aid of a singular perturbation procedure using multiple scales the coupled mode equations are derived, and Bragg reflection characteristics as a function of the magnetic field are shown numerically.

II. Analysis by a Singular Perturbation Procedure

A cross-sectional view of the gyometrical configuration and the system of coordinate used for the analysis are shown in Fig. 1(a). The permeability and permittivity of the slab have a sinusoidal variation in the y direction, a surface of the slab is grounded by a metal plate, and the biasing magnetic field H_i is applied to the z direction. Such a slab structure can be realized by arranging a ferrite slab and a dielectric slab alternatively, and some chemical resins may be used for bonding these slabs, as shown in Fig. 1(b).

In millimeter-wave frequency the permeability tensor of the ferrite medium will be nearly equal to unity. At frequency 50 GHz with the magnetic field 5 Kg the diagonal and nondiagonal components of the ferrite medium (YIG) are 1.03 and 0.1,

Manuscript received October 8, 1980; revised April 10, 1981.
M. Tsutsumi is with the Faculty of Engineering, Osaka University, Suita 565, Japan.

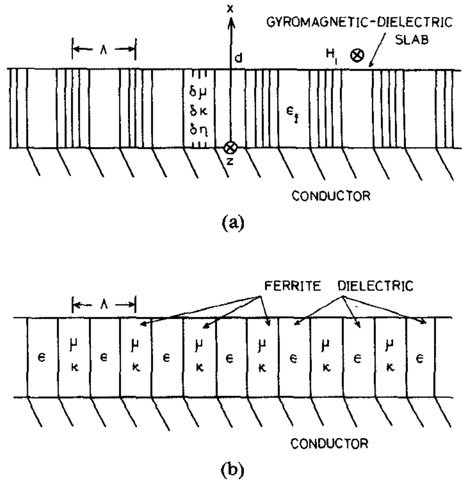

Fig. 1 System of coordinate used for analysis and structure of the periodic gyromagnetic-dielectric waveguide.

respectively [5]. Under this approximation the sinusoidal variation of the permeability tensor is assumed as

$$\hat{\mu}(y)=\mu_0\begin{bmatrix} 1+\delta\mu & -j\delta\kappa & 0 \\ j\delta\kappa & 1+\delta\mu & 0 \\ 0 & 0 & 1\end{bmatrix} \tag{1}$$

where

$$\delta\mu=\delta\bar{\mu}\cos Ky$$
$$\delta\kappa=\delta\bar{\kappa}\cos Ky$$
$$\bar{\mu}=\frac{(\gamma\mu_0)^2 MH_i}{(\gamma\mu_0 H_i)^2-\omega^2}$$
$$\bar{\kappa}=\frac{\mu_0\gamma M\omega}{(\gamma\mu_0 H_i)^2-\omega^2}$$

and $K=2\pi/\Lambda$.

δ represents a formal expansion parameter [2], [6], and [7] and Λ is the periodicity of the sinusoidal variation of the permeability. The sinusoidal variation of the permittivity is also assumed as

$$\epsilon=\epsilon_f(1+\delta\eta\cos Ky). \tag{2}$$

In the above equations it is assumed that $\delta\bar{\mu}$, $\delta\bar{\kappa}$, and $\delta\eta$ are so small that only the first-order effect of the sinusoidal variation of the refractive indexes will be taken account.

We assume that the waves do not vary in the direction of the bias field ($\partial/\partial z=0$) and that they vary sinusoidally with time and angular frequency ω, ($\exp(-j\omega t)$). The perturbed electric fields due to a sinusoidal variation of indexes can be expressed as

$$E_z=E_{z_0}(x,y_0,y_1)+\delta E_{z_1}(x,y_0,y_1)$$
$$\bar{E}_z=\bar{E}_{z_0}(x,y_0,y_1)+\delta\bar{E}_{z_1}(x,y_0,y_1) \tag{3}$$

where E_{z_0} and $\bar{E}_{z_0}$ are the unperturbed zero-order fields in the slab and the vacuum, respectively, E_{z_1} and $\bar{E}_{z_1}$ are the first-order correction terms due to the slight perturbation, and they are function of $y_0=y$ and $y_1=\delta y$ [6]. The angular frequency ω in the vicinity of the Bragg frequency ω_0 can be expanded as

$$\omega=\omega_0+\delta\omega_1. \tag{4}$$

The chain rule of the differentiation yields [6]

$$\partial/\partial y=\partial/\partial y_0+\delta\partial/\partial y_1. \tag{5}$$